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Double Inverted Pendulum stabilization Using Electric Fields and Parseval's condition

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ABSTRACT

This paper brings out a novel idea of applying the particle movement in electric fields as a precondition for stabilization of the control plant. The proposed method not only makes the system efficient due to added flexibility of changing the medium of propagation of the particle but also improvises the movement of the particle with the probability of delay being approximated and changed. The particle swarm used varies the electric field strength and thus is analyzed for influence from the particle motion and the energy efficiency is validated using parseval's condition. The integration of time delay into the system matrix enhances the controllability and improves the efficiency of the overall system in terms of speed of response, reduced energy wastage and improved settling time.

Keywords: Time delay, Double inverted pendulum, Particle swarms, Energy efficiency, Stabilization

Abbreviations: PSO – Particle Swarm optimization

1. INTRODUCTION

The application of constraints to any system is of utmost importance in understanding the feasibility for extending the range of operations of a dynamic system. Double inverted pendulum is an established model in studies on robotics, classical control and mechatronic systems. Optimizing a particular parameter for bringing in a change in the control aspects is well researched. Cycling is said to occur if the same set of variables are used in the iteration again and again. Constrained based evolution of systems would make systems predefine the conditional limits on the boundaries. This kind of approach of using constraints has been seen through researchers world-wide on typical control boundaries for dynamic systems.

The idea of a hybrid algorithm with constraints applied to a specific system where inequalities where considered and analyzed based on a fitness function approach and violation of the constraints was considered for selecting the plan of action (Jong et al.,1997). A new mathematical method was formulated with mutations from evolutionary computation, and applying the α constrained method (Takahama et al., 2013). Also, to control the convergence speed, operations on multiple simplexes, replacement of the reduction operation with the contraction operation, and the change of the algorithm parameters were introduced. We have seen developed algorithms on a master slave architecture wherein Particles in master swarm fly in the solution space by deriving information from better feasible particles, and new born infeasible particles in master swarm will guide particles in slave swarm to fly(Yang et al.,2006). A methodology was defined for set point tracking using multiple loops(Kaya et al.,2016). It has been indicated that using multiple tuning method for PID controllers, including set-point tracking, load disturbance rejection, and other requirements in the frequency and time domain(Chen et al ., 2016). Large scale unconstrained optimization, multi objective optimization with differential evolution, ε level comparison are some of the recent trends in constrained optimization which have been applied by researchers. (peng et al.,2016; Wang et al.,2016;Takahama et al.). Quasi-chaotic optimization method, Dynamic nonlinear constrained optimization problems , concept of Pareto dominance and the sequence of individual factorial design are some other interesting investigations on the constrained optimization arena (Okamoto.,2011;Liu.,2010;Fei.,2010.)

Discrete constrained optimization problems, dynamic migration strategy for enhancing the search ability of migration mechanism, rank-constrained optimization problem with a Schur-convex/concave objective function are few more varieties which have enhanced the constrained optimization.(Lukasiewycz et al.,2008;Bi.,2012;Yu et al.,2012.) The basic formulation of the dynamics of the double inverted pendulum plant were developed(kavirayani et al., 2005). Elaboration on various techniques for dealing with time delays was discussed (Sivanandam et al.,2009). Various computational techniques for analysis of complex systems were discussed (Amit 2010)and similar studies using BAT algorithm for the pendulum were developed(Nagesh et al., 2015) . A model for including the time delays into system matrix was developed for a simple mechanism (Makkapati et al., 2013) and the fundamental understanding of parseval's condition was given (kelkar et al.,1983)[20]

Thus in literature there has been extensive work in constrained based optimization applied to dynamic systems. However, there is no or little work done in the aspects of using electric field intensity and particle movements with constrained optimization used in electric fields and analyses using Parseval's condition. This paper proposes a novel method for using the particle motion under the influence of fields as an important guiding parameter.

2. MATERIALS AND METHODS

2.1. Dynamics

Lagrangian equations obtained from the Newton's second law directly give the dynamic model of a double inverted pendulum which is the plant that is used as the test bed for control related applications. The generalized coordinates were taken as (θ_1,θ_2,X) which represented the angles of the pendulum and also the cart position respectively. The derivates of these coordinates were also considered.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial \dot{q}_i} + \frac{\delta U}{\delta q_i} = Q_i \quad (1)$$

Where

L = T − U

T: kinetic energy of the pendulum system

U : potential energy pendulum system

Qi: generalized forces not taken into account in T, U

qi: generalized coordinates

Using the above equation and developing the mathematical model using kinetic and potential energies based on assumptions as in [15] a linearized model can be obtained.

The modified dynamics would involve an additional state defined from the following due to time delay τ as in [18] which is the delay induced into the system which is integrated and is calculated along with the other variables in the system matrix as in [19]. The normal initial conditions that are assumed here is that the initial position of the two pendulum is around the unstable equilibrium about the vertical position and that a small perturbation is applied about the initial condition.

2.2. Redefined methodology for optimization

Let D be the dimensional search space S ε RD

The swarm consists of N particles and ith particle is in effect in D-dimensional vector as in [17]

The position of the particle is

$$X_i = (X_{i1}, X_{i2}, X_{i3}...X_{iD})^T \varepsilon \quad S \quad (2)$$

The velocity of this particle is also a D-Dimensional vector

$$V_i = (V_{i1}, V_{i2}, V_{i3}...V_{iD})^T \varepsilon \quad S \quad (3)$$

The best previous position encountered by ith particle in S is denoted by

$$Y_i = (Y_{i1}, Y_{i2}, Y_{i3}...Y_{iD})^T \varepsilon \quad S \quad (4)$$

If Y* be the global best position among all particles, and 't' be the number of iterations involved,

During each iteration, velocity and position of each particle/swarm is updated by

$$V_i(t+1) = WV_i(t) + C_1 r_1 (Y_i(t) - X_i(t) + C_2 r_2 (Y^*(t) - X_i(t) \quad (5)$$

$$X_i(t+1) = X_i(t) + V_i(t+1) \quad (6)$$

Where $Y_i = Y_i$ if $F(X_i) \geq F(Y_i)$ (7)

else $Y_i = X_i$ if $F(X_i) < F(Y_i)$ (8)

And,

$Y^* = \{Y_0, Y_1, ...Y_n\}$ such that (9)

$$F(Y^*) = Min(F(Y_0), F(Y_1), F(Y_2), F(Y_3), ...F(Y_n)) \quad (10)$$

Where

$r_1 \approx u(0,1)$ & $r_2 \approx u(0,1)$

are uniform random sequences in range(0,1)
W is the diagonal matrix representing weight of Vi(t) as a contribution to Vi(t+1).

The constraints involved in this kind of algorithm are the that Vi is clamped to [Vmax , Vmin] to prevent PSO from leaving search space

Vmax is chosen to be K* XmaX with 0.1 ≤ K ≤ 1.0
C1 , C2 are acceleration coefficients which control the displacements of a particle in a single iteration.

2.3. Enhanced algorithm based on electric fields

Let it be considered that the particle moves in an electric field E and experiences a force F. The importance is being given here for the motion of the particle in the electric filed due to the sole reason that electric current has mainly three effects which could be either magnetic, heating or thermal and the study can be applied to interdisciplinary problems.

The motion is governed by the equation $F = QE$ Newtons as shown in figure 1.

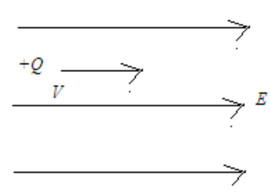

Figure 1
Particle motion in electric field

If the charge is free to move, it will receive an acceleration $a = F/m$ m/s2

Where m being the mass of the charged particle in Kg.

Assuming the search space is vaccum, the equation for update are modified with C1 & C2 are acceleration coefficients updated as

$$C_1 = C_2 = \frac{V_d}{E} \quad (11)$$

where Vd=Vi and E is assumed to be a constant field. In the absence of restraints, the velocity will increase indefinitely with time 't' provided 'E' is constant.

If the medium of propogation is gaseous or is a liquid or a solid conductors the particles collide repeatedly and loose part of their energy, however, it is assumed the movement of particles is being in space so that collisions are avoided and with constant E the average velocity or drift velocity is Vd and the drift velocity is in the direction of electric field and is given by

$V_d = \mu E$ where μ indicates the mobility. (12)

So, in this scenario, the update equations get modified as

$$V_i(t+1) = WV_i(t) + \frac{V_d}{E} r_1(Y_i(t) - X_i(t) + \frac{V_d}{E} r_2(Y^*(t) - X_i(t) \tag{13}$$

The assumption that E is being held constant would ensure that velocity increases indefinitely.

Taking into consideration the poynting vector as

$$P = E * H \tag{14}$$

Where P is interpreted as the instantaneous power density the average power density is

$$P_{xavg} = \frac{1}{2}\frac{E_m^2}{\eta} \tag{15}$$

where η is the intrinsic impedance and can be assumed to be constant for free space since

$$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 = 120\Pi \tag{16}$$

Assuming the travelling electric field E in free space is with an amplitude of 200 v/m analysis of optimizing the particle swarm is done with the aid of minimizing the power density which yields the required results for efficient convergence of the particle swarm under the influence of a constant electric field.

Effect with time delay:

The impact of time delay is considered in the update equation for the particle position update as unit time delay under normal conditions applied to the update equation Where Td and is given below as

$$X_i(t+1) = X_i(t) + V_i(t+1) * \tau_d \tag{17}$$

The impact of delay due to dead time or suffering caused by delay is directly considered during the particle position update, this way the position updates with a better accuracy.

The procedure starts with the random initialization of the cart position and velocity along with random positions of both the pendulum within five degrees span of the unstable equilibrium position. Six particles which will lead to the design of the controller gains are assumed as the particles in the population. The objective of particle swarm optimization (PSO) algorithm is to use a user defined fitness function which is square of particle position energies(SPPE) is taken as sum of all squares of particles as a Function as the fitness function where the arm angle and pendulum angle are the particles x1& x2.

$$\mathrm{Min}(x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + x_6^2 + x_7^2) \tag{18}$$

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The SPPE equation as indicated in equation (18) is obtained from equation(13) where energy of one particle was updated and now this is being updated for all the particles. The updated particle positions technically represent the moving position of the states that govern that particular dynamic system. For instance, the time delay is one state which is moving in the state space along with the other three states of the position and angular position of the pendulums.

2.4. Integrated Analysis Parsevals' theorem

The energy of a casual system F(t) is given by the integral of the square of the function over infinite time period provided that the signal energy is finite and the signal is absolutely integrable, i.e.

$$\int_0^\infty f^2(t)dt = \frac{1}{2*\pi*j}\int_{-j\infty}^{j\infty} F(s)F(-s)ds$$

The definite integral for continuous time integrals can be evaluated using the following expression

$$J_n == \frac{1}{2*\pi*j}\int_{-j\infty}^{j\infty} \frac{B(s)B(-s)}{A(s)A(-s)}ds$$

Where J1 and J2 are defined from the basic equation for a system transfer function defined by

$$F(s) = \frac{N(s)}{D(s)}$$

Where $N(s) = b_0 + b_1 s + \cdots \dots \dots .. b_{n-1}S^{n-1}$

And $D(s) = a_0 + a_1 s + \cdots \dots \dots .. a_n S^n$

Yielding $J_1 = \frac{b_0^2}{2a_0a_1}$ and $J_2 = \frac{b_1^2 a_0 + b_0^2 a_2}{2a_0a_1a_2}$

Thus taking this analysis for the present plant model, the various values for the system matrices are as follows. The various values obtained for the double inverted pendulum system are shown in table 1.

Table 1

Parseval's condition data from transfer function defintions

Parameter	**Values**	**J1**	**J2**
Cart Position	b0=578.2;b1=5.98*10-3 a0=0;a1=0	∞	∞
Cart Velocity	b0=578.2;b1=0 a0=0;a1=1700	∞	∞
Lower Pendulum	b0=58.99;b1=8.9*10-16 a0=1700;a1=0	∞	∞
Upper Pendulum	b0=578.2;b1=0 a0=0;a1=1700	∞	∞

Thus based on the values defined in the table and the values obtained for J1 and J2 it can easily be concluded that the energy being used in this system is not at the optimal value and that it requires the energy to be minimized. If Jn is the energy and is to be minimized for each of the transfer function then the value of energy can be optimized by pole-zero cancellation resulting in a reduced value of J1 and J2 . The reduction of the energy usage can be achieved by using one of the generally used integral performance criteria such as the integral absolute error criteria, integral square error criterion where the equations are as follows

$$J_i^1(\varphi) = \int_0^{\infty} t^i |e(\varphi)t| dt \quad \forall\, i = 0,1,2$$

Which is the general integral absolute error criterion which requires to be minimized. Or, the performance criterion could be

$$J_i(\varphi) = \int_0^{\infty} (t^i e(\varphi)t)^2 dt \qquad \forall\, i = 0,1,2$$

Where ϕ denotes variable parameter which are chosen to minimize Ji(ϕ). The advantage of such a process will yield the following

If a control system is designed to minimize J0, then the response to a step has a relatively high overshoot. This overshoot can be decreased by using a higher value of i and response for i=1 optimization are often quite similar for both the performance indices Ji and J11

The choice of i=1 in Ji which is known as the ISTE criterion usually gives satisfactory results.
Process identification is essential for control, analysis diagnostics and fault accommodation in system.

Thus by placing constraints on the possible values that can be taken for overshoot and settling time, the values of J can be altered for the integrated model which is achieved here with the help of the fuzzy control which operates with a controller designed taking into consideration the aspects of boundary conditions.

3. RESULTS

The experiments were conducted in standard MATLAB environment through scripting and the results are directly extracted from the automatic plotting facility. A particle swarm optimized with constraints would yield the following results as shown in figure 2 and 3.The figures were obtained using MATLAB software simulation environment. Taking the best particles resulting from the optimization routine, the particles are directly used in the pole placement design and are compared with the standard LQR algorithm. Figures 4 to 9 indicate the variations in the output states

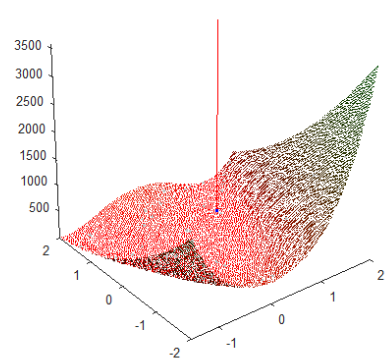

Figure 2
3D representation of particle space

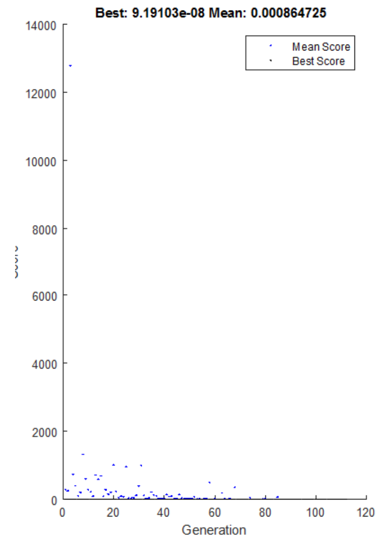

Figure 3

Particle score with generations

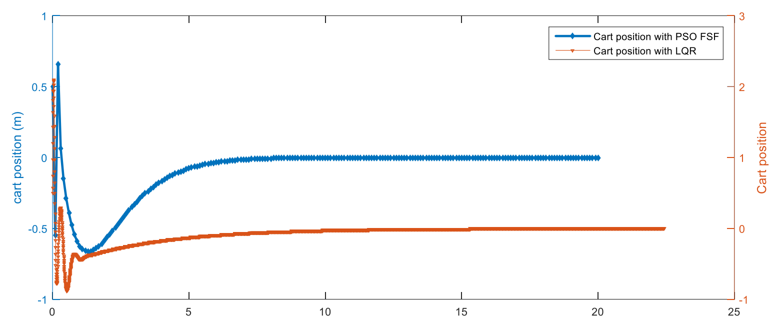

Figure 4

Cart position variation with Particle swarm compared to LQR

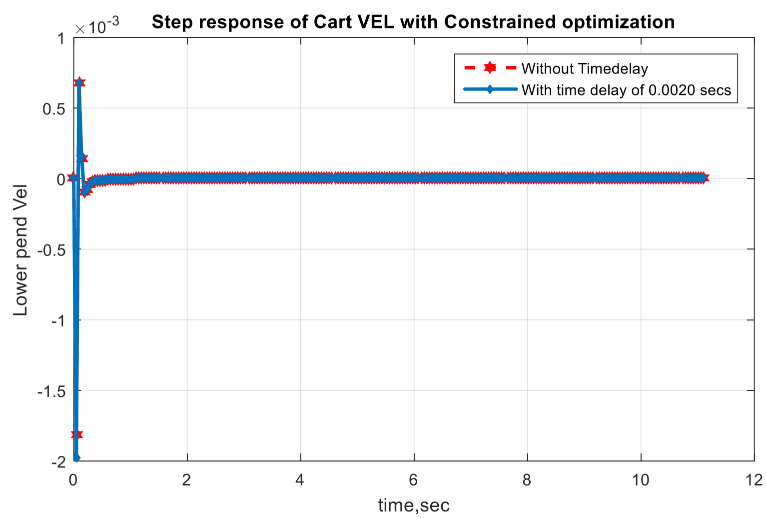

Figure 5

Comparision of cart velocity variation with constrained optimization

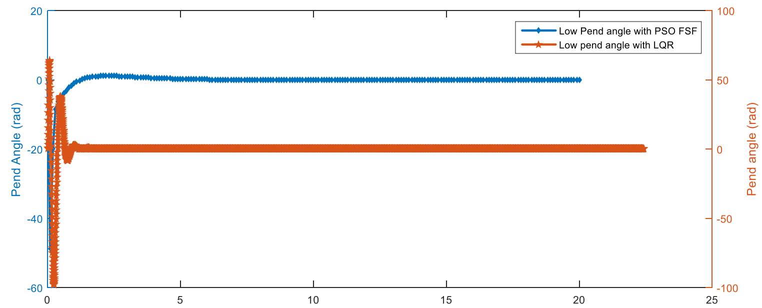

Figure 6
Lower pendulum variation

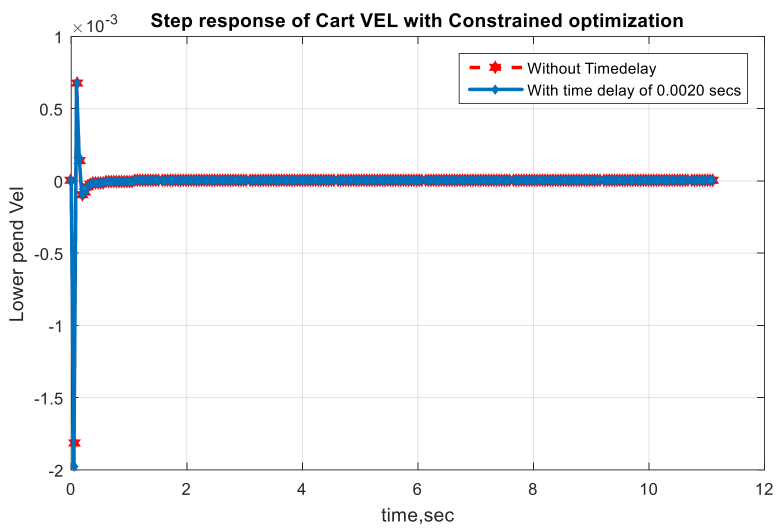

Figure 7
Cart velocity variation

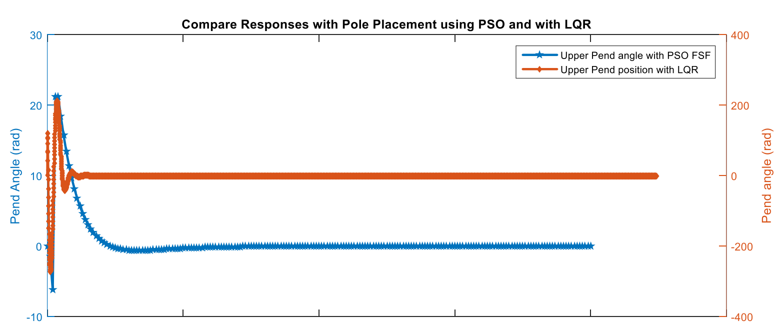

Figure 8
Upper pendulum variations

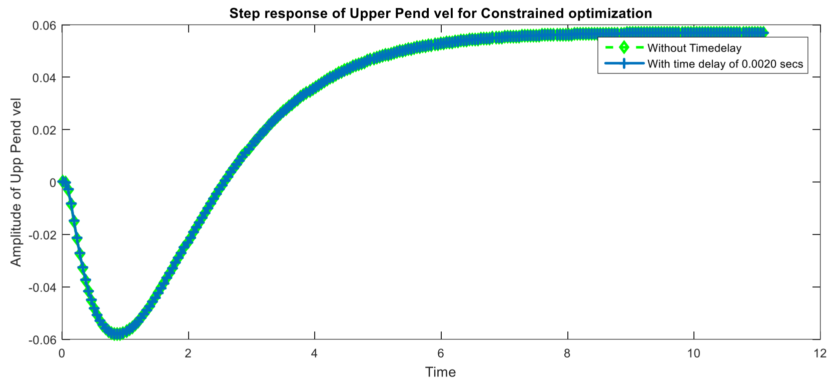

Figure 9

Upper pendulum velocity variation

4. DISCUSSION

From the results it appears that the parseval's condition based studies would help design an energy efficient controller for the plant. The use of the particle swarm and fine tuning the particle motion using the particle swarm and energy field definition for the governing fitness function will help in a more controlled and constrained optimization on the end result requirement of the plant model. The following important observations are raised from the discussed model

- The plant model is enhanced with row and column generation by integrating the time delay model into the plant dynamics which makes it a unique representation of the plant with minimum set of variables defining the linearized plant model with delays embedded into the system.
- The particles definition in PSO has evolved with the definition of electric field parameters and this makes it a constrained optimization which yields better results when compared to classical PSO without constraints.
- The use of Parseval's condition is a final test on the system dynamics and energy usage which indicates based on the system that it is energy efficient design.

5. CONCLUSION

It can be concluded from the analysis that the particle swarm optimization method developed using electric fields as a guiding parameter yielded better results in terms of energy efficiency as the algorithm is custom designed for particle movement in electric field. The variation of various output states indicates that the control effort is relatively less when a constrained optimization method is used for the system when compared to unconstrained optimization.

SUMMARY OF RESEARCH

1. The reason for choice of the control bed of double inverted pendulum is that it is an under actuated system
2. Particle swarm optimization was chosen for the control strategy in view of the advantages of the PSO when compared with other naturally inspired algorithms, considerably treating PSO as the mother of all the naturally inspired algorithms
3. Time delay component was added in view of the criticality involved in studies when the system has to respond to time delays. Thus row and column addition in both the system matrix and input matrix defined a unique system which was analyzed.
4. Parseval's theorem was used to analyze the system for energy efficiency in view of the vivid advantages involved.

FUTURE ISSUES

I believe that many scientists in sphere of control domain can use the developed methodology for extending studies on integrated models by including the time delay component into the system matrix and using the electric field extension for the analysis of the particle swarm optimization.

DISCLOSURE STATEMENT

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